Naval Research Laboratory
Washington, DC 20375-5320

NRL/MR/6700--01-8542

# POD (K-L) Analysis of the Interaction Free Dynamics of a Beam/Pendulum Configuration: An Experimental Study

IOANNIS T. GEORGIOU

*Department of Naval Architecture and Marine Engineering*
*National Technical University of Athens*
*Athens, Greece*

IRA B. SCHWARTZ

*Special Project for Nonlinear Science*
*Plasma Physics Division*

April 30, 2001

# REPORT DOCUMENTATION PAGE

*Form Approved*
*OMB No. 0704-0188*



| 1. AGENCY USE ONLY (*Leave Blank*) | 2. REPORT DATE | 3. REPORT TYPE AND DATES COVERED |
|---|---|---|
| | April 30, 2001 | Interim |

4. TITLE AND SUBTITLE

POD (K-L) Analysis of the Interaction Free Dynamics of a Beam/Pendulum Configuration: An Experimental Study

5. FUNDING NUMBERS

6. AUTHOR(S)

Ioannis T. Georgiou* and Ira B. Schwartz

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

Naval Research Laboratory
Washington, DC 20375-5320

8. PERFORMING ORGANIZATION REPORT NUMBER

NRL/MR/6700--01-8542

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

Office of Naval Research
Arlington, VA 22217

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

*Department of Naval Architecture and Marine Engineering, National Technical University of Athens, Athens, Greece

12a. DISTRIBUTION/AVAILABILITY STATEMENT

Approved for public release; distribution is unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT (*Maximum 200 words*)

Here we present an analysis of the experimental free dynamics of beam/pendulum configuration. We apply the method of Proper Orthogonal Decomposition. The method identifies the dominant dynamics. Depending on the level of energy a motion is initiated, the interaction dynamics bear distinct signatures of the slow reduced dynamics and distinct signatures of the interaction dynamics.

14. SUBJECT TERMS

| | | |
|---|---|---|
| Nonlinear dynamics | Computational mechanics | Chaos |
| Resonance | Continuum mechanics | |

15. NUMBER OF PAGES

10

16. PRICE CODE

| 17. SECURITY CLASSIFICATION OF REPORT | 18. SECURITY CLASSIFICATION OF THIS PAGE | 19. SECURITY CLASSIFICATION OF ABSTRACT | 20. LIMITATION OF ABSTRACT |
|---|---|---|---|
| UNCLASSIFIED | UNCLASSIFIED | UNCLASSIFIED | UL |

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std 239-18
298-102

# POD (K-L) ANALYSIS OF THE INTERACTION FREE DYNAMICS OF A BEAM/PENDULUM CONFIGURATION: AN EXPERIMENTAL STUDY

**Ioannis T. Georgiou**
Department of Naval Architecture and Marine Engineering
National Technical University of Athens
Athens, Greece

**Ira. B. Schwartz**
Special Project for Nonlinear Science
Code 6700.3,Plasma Physics Division
Naval Research Laboratory
Washington, DC 20375
Email: schwartz@nlschaos.nrl.navy.mil

## ABSTRACT

Here we present an analysis of the experimental free dynamics of beam/pendulum configuration. We apply the method of Proper Orthogonal Decomposition. The method identifies the dominant dynamics. Depending on the level of energy a motion is initiated, the interaction dynamics bear distinct signatures of the slow reduced dynamics and distinct signatures of the interaction dynamics.

## INTRODUCTION

The dynamics of coupled structural systems that consist of linear elastic continua coupled to nonlinear oscillators or other nonlinear continua is of interest to the structural engineering community for at least two reasons: First, structures and machines in applications are by design geometrically complicated configurations of linear and nonlinear structural elements. And second, attachment of nonlinear oscillators or layers of nonlinear material can modify in profound ways the resonant dynamics of large-scale structures. In designing geometrically complicated structures such as an aircraft fuselage or a ship to avoid interaction resonant dynamics between the main structure and the attached machinery is a basic issue. Depending on the strength of coupling between primary and secondary substructures, the dynamics of coupled systems may be as simple as periodic in time and space and as complex as chaotic in both space and time. A measure of complexity is the number of active degrees-of-freedom that supports a motion. The basic issue is to identify these degrees-of-freedom and relate them to normal modes of oscillation. An understanding in depth could be obtained if the complexity is related to the complexity of the dynamics of the uncoupled substructures.

A way to compute and understand complexity of coupled linear/nonlinear systems is to consider first the degree of complexity for the case of weak coupling among its various substructures. In general, a geometrically complicated coupled structure is an assembly of primary and secondary substructures. For instance a soft/stiff-coupled system is composed of substructures of low natural frequencies (secondary) and substructures of high natural frequencies (primary). In this case, we have weak coupling.
This approach allows the usage of ideas from geometric singular perturbation theory to relate the reduced dynamics to the dynamics of the uncoupled substructures. In particular, for sufficiently weak coupling, the free dynamics of coupled systems can be separated into slow and fast dynamics.
The dynamics of a coupled system with weak coupling are governed by the slow dynamics, which manifests itself as a number of normal modes of oscillations or in phase space as two-dimensional invariant manifolds. Transversely to this subspace of slow dynamics reside the fast dynamics. *Dynamics that are not restricted in the fast and slow subspaces are termed interaction dynamics. Interaction dynamics are developed due to bifurcation mechanisms that destabilize the slow and fast reduced dynamics. Such a mechanisms is global internal resonance that takes place at high energy levels.*

The spatio-temporal complexity of the dynamics depends on the strength of coupling between the subsystems and the complexity of the dynamics of the uncoupled subsystems. The complexity has to do with the new dynamics that are developed due to instabilities of the reduced slow and fast dynamics. Towards developing a systematic understanding of the dynamics coupled linear/nonlinear systems, we have considered the dynamics of a beam/pendulum coupled system.

Manuscript approved March 26, 2001.

This is a prototype for coupled linear/nonlinear structures. Aspects of its dynamics have been studied in detail numerically and experimentally by combining the methods of geometric singular perturbation and the computational method of proper orthogonal decomposition (POD) or K-L. In this work we determine by the POD method the degrees-of-freedom that support the interaction free dynamics of the beam/pendulum-coupled system.

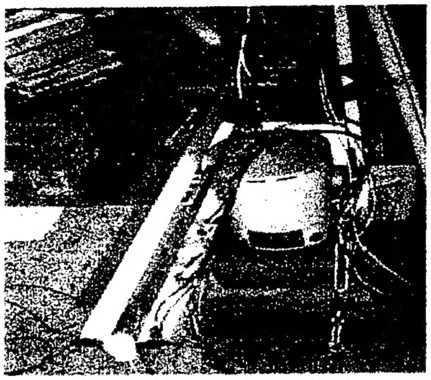


**Figure 1 Experimental set-up of the beam/pendulum configuration.**

## MEASUREMENT OF EXPERIMENTAL DYNAMICS

A coupled system composed of a two-branched cantilevered aluminum beam supporting at its free end a pendulum, shown in Figure 1, was build and tested. The length, width, and thickness of the beam were respectively 660.40 mm, 25.40 mm, and 3.175 mm. The length and mass of the pendulum were varied to achieve coupling strength of various levels since the coupling is determined by the freq uency ratio and beam ratio defined respectively by

$$\mu = \frac{\omega_p}{\omega_b}, \quad \beta = \frac{M_p}{M_b},$$

where $\omega_p$ and $\omega_b$ are the linear natural frequency of the uncoupled pendulum and the fundamental frequency of the beam. Moreover, $M_p$ and $M_b$ are the corresponding masses. A typical experiment was performed as follows: The pendulum mass was released from a specified initial angular position while the beam was at rest. After the elapse of a time interval of three minutes, acceleration data at seven locations of the beam were recorded.. Table 1 shows the corresponding coupling parameters and initial conditions for a set of four experiments. Table 2 gives the locations and sensitivities of the accelerometer sensors

**Table 1 Initial conditions and coupling parameters**

| Beam/Pendulum Configuration $L_b = 660.40mm$, $M_b = 365gr$, $\omega_b = 28.525\ rad/s$ | | | |
|---|---|---|---|
| Experiment | Initial conditions | Coupling | |
| *No.* | $\theta_0$ | $\mu$ | $\beta$ |
| 1 | $\pi/2$ | 0.2025 | 0.050 |
| 2 | $\pi/2$ | 0.2100 | 0.100 |
| 3 | $\pi$ | 0.2100 | 0.100 |
| 4 | impact at $\theta_0 = \pi$ | 0.2100 | 0.100 |

**Table 2 Spatial arrangement of accelerometer sensors**

| Accelerometer sensors | | |
|---|---|---|
| Number | Location/beam length | Sensitivity ($mV/g$) |
| 1 | 0.1763 | 09.84 |
| 2 | 0.3110 | 09.72 |
| 3 | 0.4443 | 96.60 |
| 4 | 0.5790 | 98.10 |
| 5 | 0.7138 | 09.80 |
| 6 | 0.8471 | 10.15 |
| 7 | 0.9773 | 10.12 |

Notice in Figure 1 that the beam consists of two branches coupled by a pin about which the pendulum rotates. The accelerometers are attached along one branch. This clearly disturbs the symmetry of the structure. This would create some discrepancies in comparing the experimental results with the analysis of the idealized beam/pendulum problem. The identification power of the POD method resides into the fact that it can detect these differences. In fact, this is the case in this experiment.

A way to compute how complexity evolves is to initiate motions at different energy levels and analyze them by the method of proper orthogonal decomposition as discussed below.

The figures below show the free o-temporal free response of the beam as measured by the accelerometers.

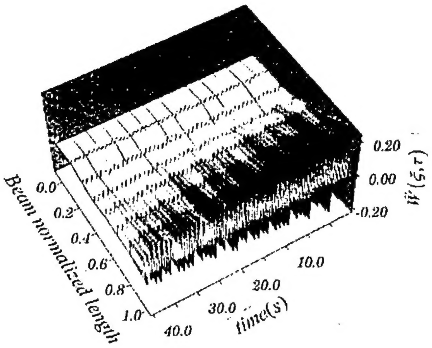



**Figure 2 Experiment No. 1**

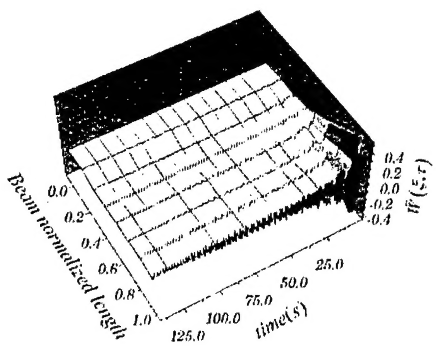



**Figure 3 Experiment No. 2**

Although the initial conditions are the same and the difference in strength of coupling is small, for the behavior seems to quite different. In fact, shall see that more degrees of freedom support the motion shown in Figures 3,4,5.

## POD ANALYSIS OF THE EXPERIMENTAL DATA

We are interested in computing from the experimental data the optimum degrees-of-freedom that dominate the free dynamics for the experiments listed in Table 2. Processing the data by the method of Proper Orthogonal Decomposition (POD) or K-L will identify the spatio-temporal characteristics of these degrees-of-freedom. It is convenient, to represent the experimental time series as a vector:

$$\ddot{\vec{W}}(t) = \left(\ddot{W}_0(t), \ddot{W}_1(t), \cdots, \ddot{W}_7(t)\right)^T, \quad t \in [T_1, T_2].$$

This vector of measurements is written as a fluctuation about a its average value, that is,

$$\vec{W}(t) = \vec{w}(t) + \left\langle \ddot{\vec{W}}(t) \right\rangle, \quad \left\langle \ddot{\vec{W}}(t) \right\rangle \equiv \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} \ddot{\vec{W}}(t)\, dt$$

Its fluctuation will be expanded into proper orthogonal modes:

$$\vec{w}(t) = \sum_{m=1}^{M} \sqrt{\lambda_m} A_m(t) \vec{\Phi}_m$$

This process of measurements is called a Proper Orthogonal Decomposition analysis. The constant $\lambda_m$ represent the fraction of energy contained in the mode whose amplitude and shape are given respectively by the functions $A_m(t)$ and $\vec{\Phi}_m$. The set

$$\Lambda \equiv \left\{\lambda_m\right\}_{m=1}^{m=M}$$

gives the spectrum of energy distribution over the POD modes. Clearly since it shows the number of active degree-of-freedom, it is a very useful quantity of a motion.

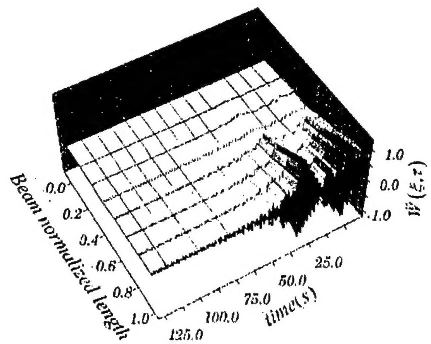



**Figure 4 Experiment No. 3**

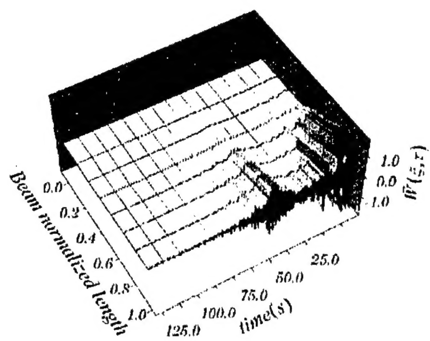



**Figure 5 Experiment No. 4**

**POD Analysis of Noise.** Since we deal with experimental data it is useful in evaluating the results of the POD analysis to perform a POD analysis of a spatio-temporal record of noise measurements. The noise signal shown in Figure 6 was processed by the POD method. Figure 7a shows that its POD spectrum is uniformly distributed over a number of POD modes that is equal to the number of site measurements. Figures 7b and 7c show respectively the time history and frequency spectrum of the amplitude of the first POD mode of noise.

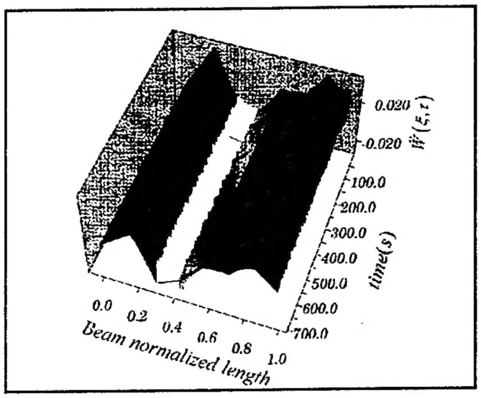



**Figure 6 Spatio-temporal measurement of noise**

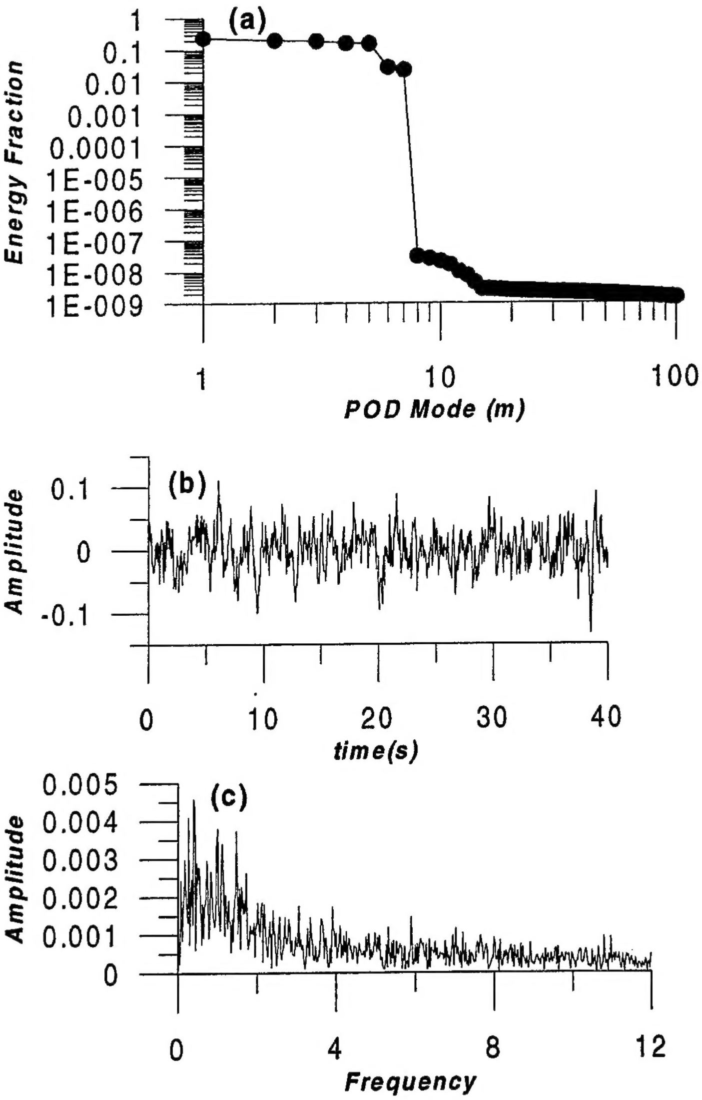



**Fig. 7 POD analysis for noise: (a) spectrum, (b) time series of first POD mode, (c) FFT of first POD mode.**

The POD analysis of the noise database will help interpret the results of the POD analysis of the spatio-temporal measurements of the free dynamics of the beam/pendulum coupled system.

**Case A:** $\mu = 0.2025$, $\beta = 0.050$. At sufficiently weak coupling it is possible to have pure slow dynamics. These dynamics are slow oscillations and form in phase space a two-dimensional invariant manifold, the geometric picture of a global nonlinear normal mode. This normal mode is divided into the master sub-mode, which is a perturbation of the uncoupled pendulum dynamics, and the slaved sub-mode, which is associated with the beam substructure. The slow-slaved dynamics of the beam substructure have the characteristic frequency signature [1]:

$$F_s = \{2\omega_p, 4\omega_p, 6\omega_p, \cdots\}$$

, whereas that of the master slow dynamics is given by:

$$F_m = \{\omega_p, 3\omega_p, 5\omega_p, \cdots\}$$

The slaved slow dynamics can destabilize at high energy levels. These destabilization mechanics, one of which is a global internal resonance [Georgiou et. al. 1999], create the interaction dynamics, that is, dynamics qualitatively different than small regular perturbations of the dynamics of the uncoupled subsystems.

For experiment No. 1, the value of coupling level is not sufficiently small, but sufficiently large for the interaction dynamics due to instabilities of the slow dynamics to occur. The goal of this experiment is to identify the first signs of interaction dynamics at weak coupling. The pendulum is released from an angle slightly greater than $\pi/2$ radians. Data were collected after the elapse of three minutes, sufficient time for high frequency transients to die out. The spatio-temporal measurement of the beam acceleration is shown in Figure 2. Figure 8a reveals that its spectrum is concentrated at one mode. The plateau looks similar to that of noise. Indeed, all POD modes but the first one are noise. The amplitude dynamics of the first mode is shown in Figure 8b whereas it's FFT in Figure 8c. We notice how noise free is the time history of the single POD mode. The frequency spectrum of the POD mode contains not only the fundamental frequency of the beam but also lower and higher frequencies, which clearly have nothing to do with the natural frequencies of the beam. These frequencies are multiples of the pendulum frequency. In fact the frequency spectrum of the POD mode turns out to be

$$F_b = \{2, 4, 4.9485, 6.1067, 6.9489\} \times \omega_p$$

This is an interweaving distortion of the frequency signature of the slaved slow and master dynamics. The shape of the POD mode is shown in Figure 8d. It is identical to the shape the beam takes when the system executes pure slow motions.

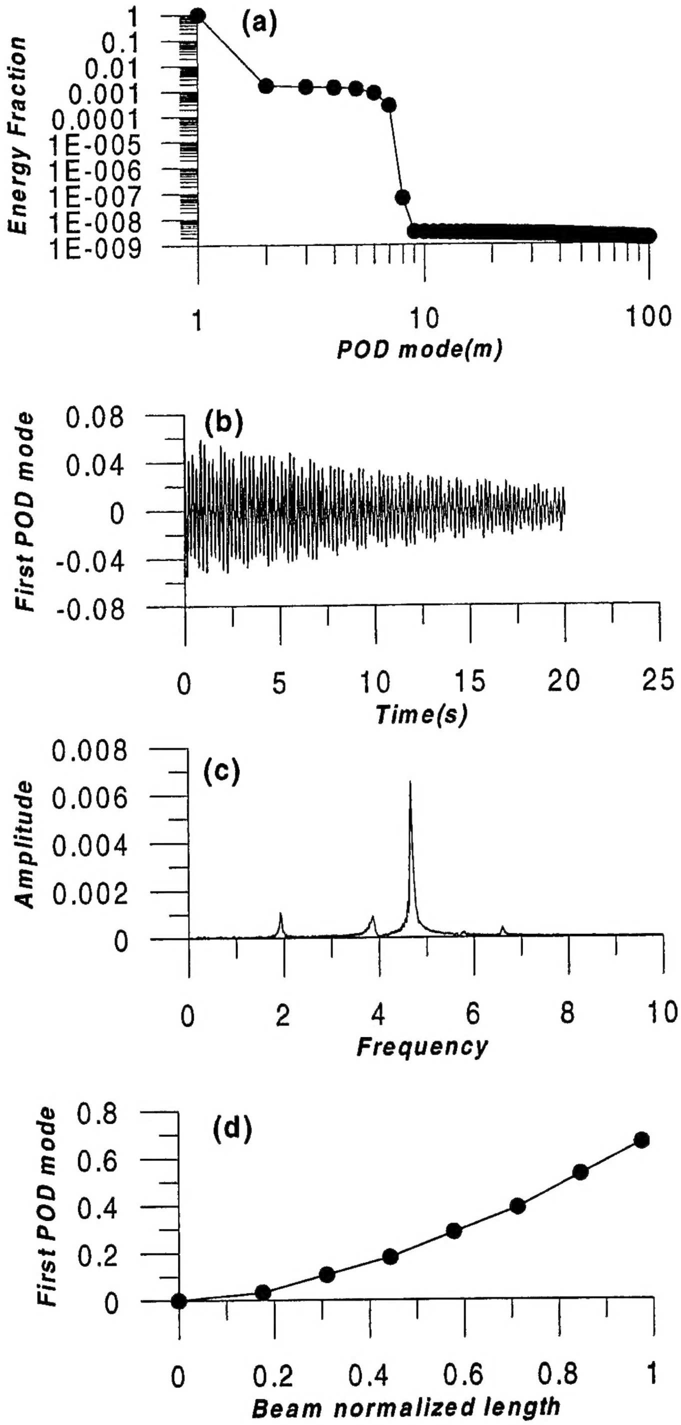



**Fig. 8 POD analysis: (a) spectrum, (b) amplitude of first mode, (c) FFT of first mode, (d) shape of first mode.**

__Case B:__ $\mu = 0.2100$, $\beta = 0.100$. To explore how the interaction dynamics change as a function of energy level, we have performed three experiments at different energy levels at a slightly higher value of coupling than in case A. The spatio-temporal measurements are shown in Figures... The initial conditions are shown in the Table 1. Figure 9 shows the POD spectra for these experiments. Now the energy is distributed over 2-3 modes. However, one mode dominates. All modes in the plateau have very small energy and are noise.

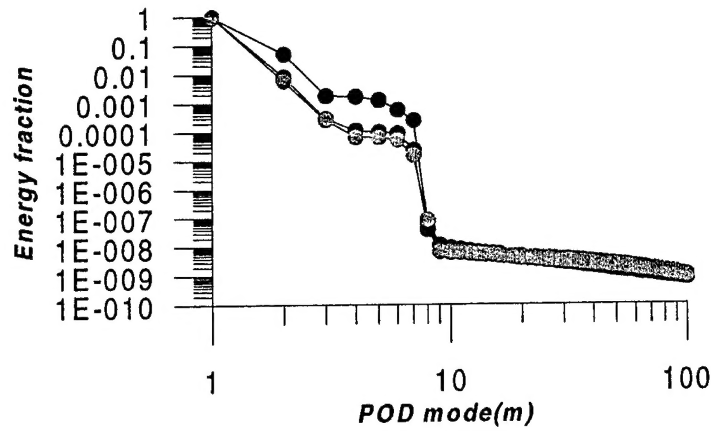
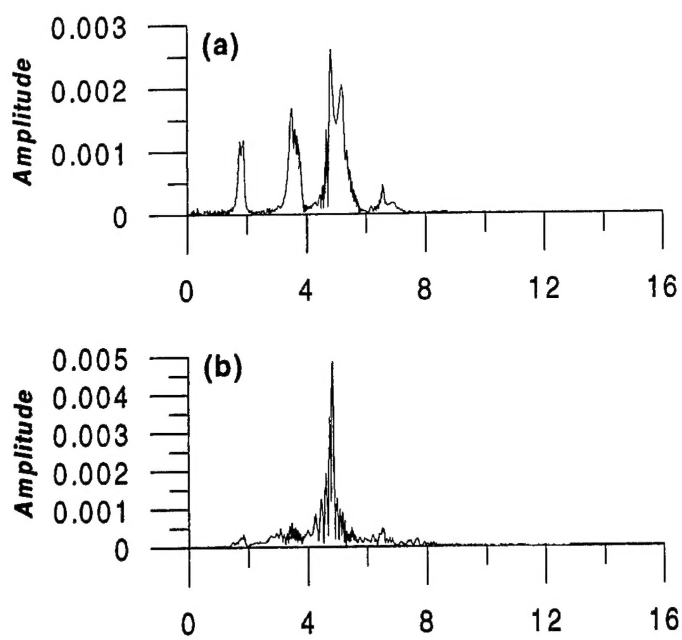



**Fig. 9 POD spectrum for experiments 2, 3, and 4.**

Figure 10 shows the frequency spectrum of the first POD mode. Here we see how the frequency content changes as the energy level in increased. One frequency remains unaffected by the energy lever. This frequency is related to the fundamental frequency of the beam. It is interesting to see the dramatic change the frequency spectrum undergoes as the frequency of the pendulum approaches zero or the pendulum performs a near heteroclinic motion. However, the shape of this mode does not change as we can see in Figure 11



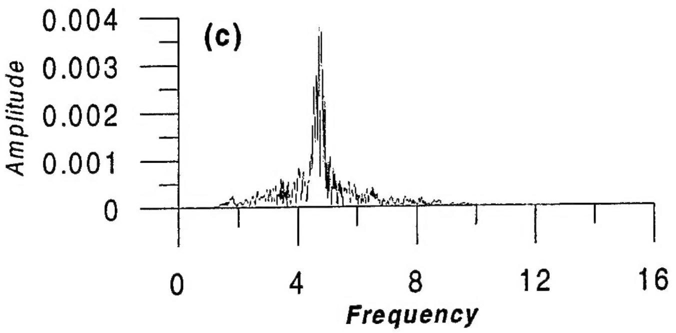



**Fig. 10 POD analysis of experiment no. 2**

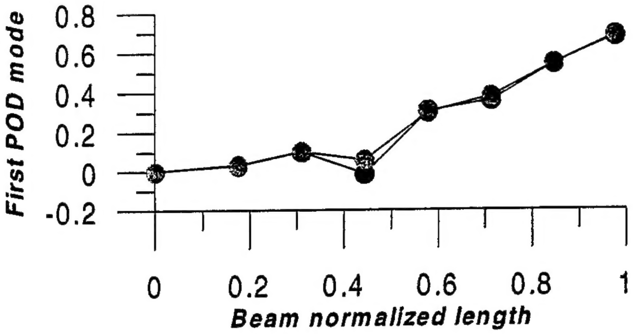
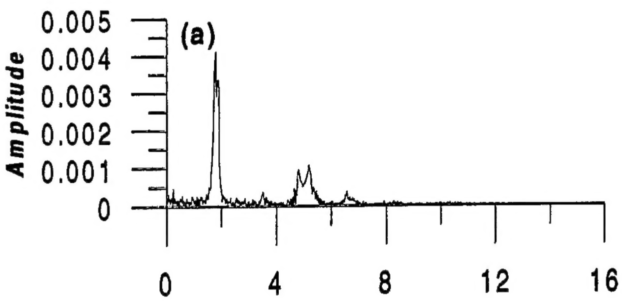
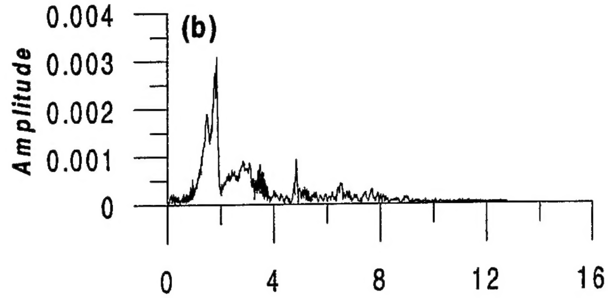



**Fig. 11 Shape of first POD mode**

At sufficiently weak coupling the slow dynamics are characterized by a slow invariant manifold or normal mode. This mode is described by two POD modes whose beam component coincides, contain different energy fractions, with the shape of the fundamental mode of the beam. The shape here remains almost the same. Thus the question *of whether the POD mode is somehow related to a normal mode arises naturally.*



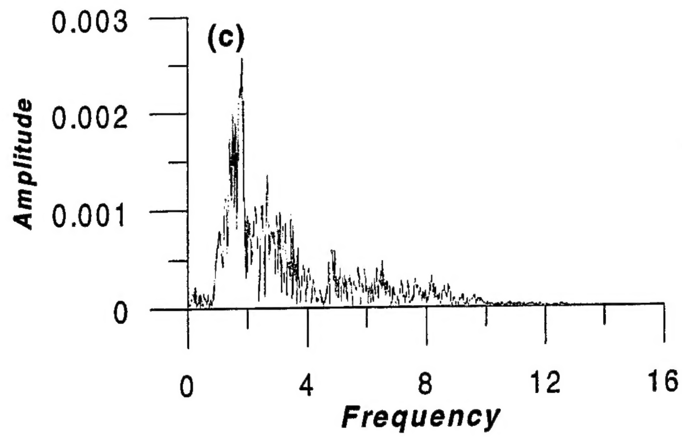




**Fig. 12 FFT for the second POD mode**

Figure 12 shows the frequency spectrum of the second POD mode of all three experiments. They are distributed around a low frequency that remains almost the same for energy levels. This turns out to approximately twice the natural frequency of the pendulum. We that clearly in Figure.. for the case of small coupling. These modes have almost identical shape as we can see in Figure 13. This shape is localized and clearly does not resemble to the shapes of the modes of the uncoupled beam. This is a product of the interaction. This could be used as diagnostic for the structural integrity of the linear structure.

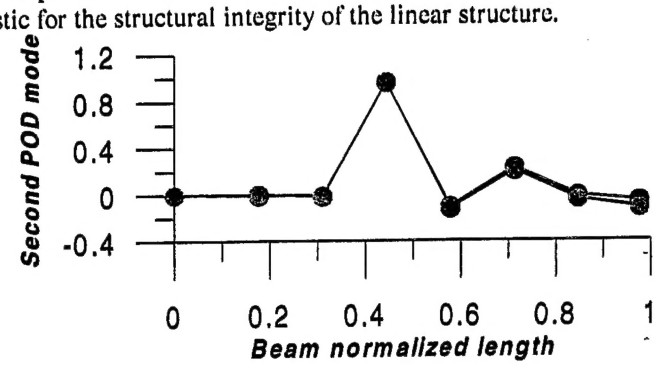



**Fig. 13 Shape of second POD mode**

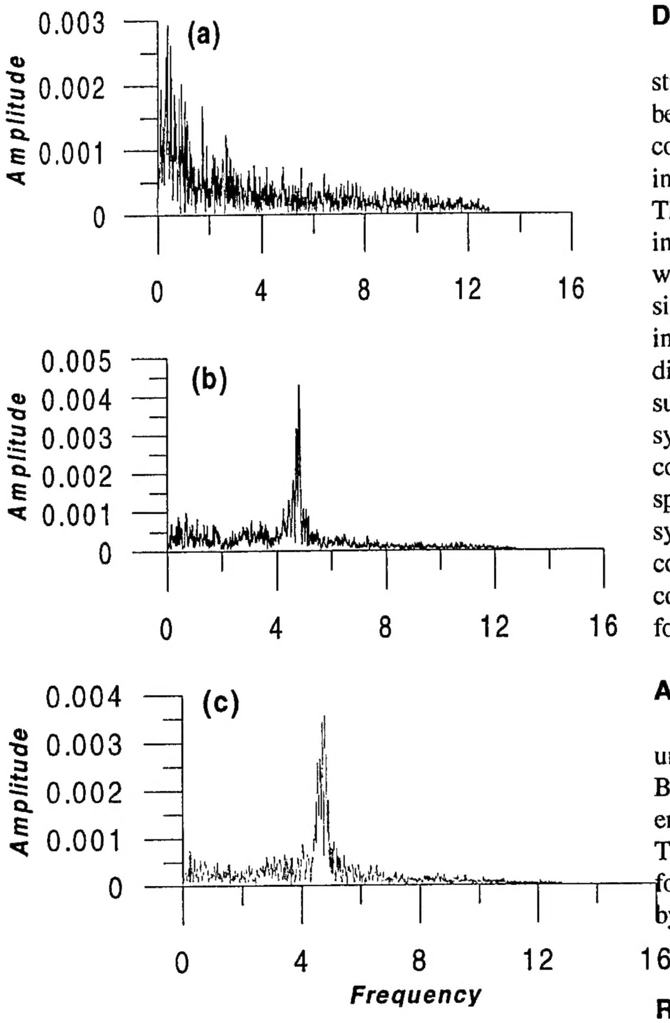



**Fig. 14 FFT of the third POD mode.**

Figure 14 shows the frequency spectra of the third POD mode. For the first experiment, this mode is noise. For the other two experiments, the frequency spectra are characterized by a frequency, which is almost twice that of the natural frequency of the pendulum. These two modes have identical shapes as shown in Figure 15.

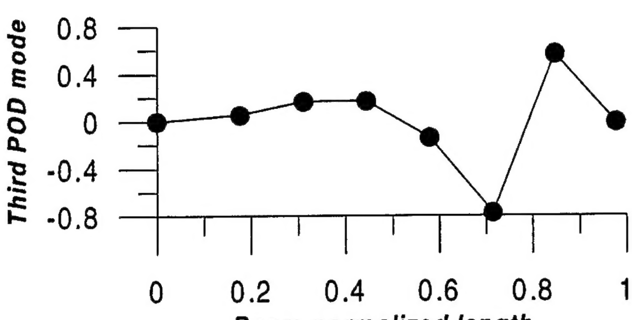



**Fig. 15 Shape of the third POD mode.**

## DISCUSSION

The POD processing of experimental dynamics of coupled structures is a powerful computational tool. In particular, it can be used effectively to process the interaction free dynamics of coupled structures. This tool is very useful to understand the interaction dynamics of linear/non-linear coupled structures. The interactions dynamics have their roots to the transversal instabilities of the reduced slow and fast dynamics of the weakly coupled system. This study here showed that a distorted signature of the slaved slow dynamics is carried by the interaction dynamics. This distorted frequency signature is a diagnoses of interaction between the main and primary substructures of the coupled system. We believe that a systematic study of the interaction dynamics as a function of coupling will reveal universalities regarding the build up of spatial and temporal complexity in high dimensional coupled systems whose parts form a complicated configuration. In a coming work we compare the finding of this experiment to computational results from a simulation of the system formulated as a singular perturbation problem.

## ACKNOWLEDGMENTS

The experiment was performed at the dynamics laboratory, under the direction of Prof. Alexander Vakakis and Prof. Berkman, at the department of mechanical and industrial engineering of the University of Illinois at Champaign Urbana. The authors thank Prof. Alexander Vakakis and Mr. E. Emaci for their help with the experiment. This work was supported by the Office of Naval Research.

## REFERENCES

Georgiou, I. T, Schwartz, I. B., Emaci, E., and Vakakis, A. F., 1999, "Interaction Between Slow and Fast Oscillations in an Infinite Degree-of-Freedom Linear System Coupled to a Nonlinear Subsystem: Theory and Experiment," *Journal of Applied Mechanics*, Vol. pp. 448-459.

Georgiou, I. T., Schwartz, I. B., 1999, "Dynamics of Large Scale Coupled Structural/Mechanical Systems: A Singular Perturbation/Proper Orthogonal Decomposition Approach," *J. App. Math. (SIAM)*, Vol. 59, No. 4, pp. 1178-1207.

Jones, C., 1995, *Geometric singular perturbation theory in dynamical systems* (Spring Lecture Notes Math. 1609), pp. 44-120.

. Cusumano, J. P., Sharkady, M. T., and Kimple, B. W., 1994, "Experimental measurements of dimensionality and spatial coherence in the dynamics of a flexible-beam impact oscillator," *Phil. Trans. R. Soc. Lond.*, Vol. A347, pp. 421-438.

Sirovich, L., 1987, "Turbulence and the Dynamics of Coherent Structures, Pt. 1: Coherent Structures," *Quart. Appl. Math.*, Vol. 45, pp. 561-571.

Georgiou, I. T., and Schwartz, I. B. 1999, "The Slow Nonlinear Normal Mode and Stochasticity in the Dynamics of

a Conservative Flexible Rod/Pendulum Configuration," *Journal of Sound and Vibration*, Vol. 220, No. 3, pp. 383-411.

Ma X. and Vakakis, A. F. 1999, "Karhunen-Loeve Decomposition of the Transient Dynamics of a Multibay Truss," *AIAA Journal*, Vol. 37, No. 8, 1999, pp. 939-946.

Georgiou, I. T., and Schwarz, I. B. 1999c,"The Slow Nonlinear Normal Mode and Stochasticity in the Dynamics of a Conservative Flexible Rod/Pendulum Configuration," *Journal of Sound and Vibration*, Vol. 220, No. 3, pp. 383-411.